



# 21st Annual GIS/CAMA Technologies Conference Chattanooga Convention Center

GIS/CAMA • Chattanooga, TN



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March 6-9, 2017

# Continuing Education (CE) Credit

Recertification Credit forms for CE credit can be collected from the Registration Desk on Thursday.

## Housekeeping

The conference proceedings will be available approximately 8 weeks after the conference.





# Vertical Equity Decathlon: PRD vs. PRB

By: Josh Myers

Josh Myers Valuation Solutions



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# The Greatest All-Time Decathlete: Ashton Eaton



Background: Where are we and how did we get here?



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# Comparison of PRD and PRB According to Ten Factors



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# Ease of Calculation

- **Coefficient of Price-Related Bias (PRB)**

Regression:  $(R - \text{median}(R)) / \text{median}(R) = b_0 + b_1 * \log_2(\text{Value\_Proxy}) + E$ ,  
where  $\text{Value\_Proxy} = 0.5 * (\text{AV} / \text{median}(R)) + 0.5 * \text{SP}$ ,  $R = \text{AV}/\text{SP}$ , and  $E =$   
error. The Coefficient of Price-Related Bias is  $b_1$ , the slope coefficient.

- **Price Related Differential (PRD) - Traditional**

= mean ratio / weighted mean ratio

=  $\text{mean}(R) / (\text{mean}(R * \text{SP}) / \text{mean}(\text{SP}))$

=  $\text{mean}(R) / (\text{mean}(\text{AV}) / \text{mean}(\text{SP}))$

=  $\text{mean}(R) / (\text{sum}(\text{AV}) / \text{sum}(\text{SP}))$

=  $(\text{mean}(R) * \text{sum}(\text{SP})) / (\text{sum}(\text{AV}))$

where  $R = \text{AV}/\text{SP}$



# Ease of Calculation

- Both measures require a computer for calculation.
- However, the PRD's formula is simpler than the multi-step regression calculation entailed as a part of the PRB, and so will be easier for most practitioners to run in a program like Excel.





# General Assumptions

- The PRB is reliant upon the standard set of least squares linear regression assumptions. The PRD is not.
- Also, the PRD does not require the User to understand anything about regression analysis, because it uses common statistical functions.



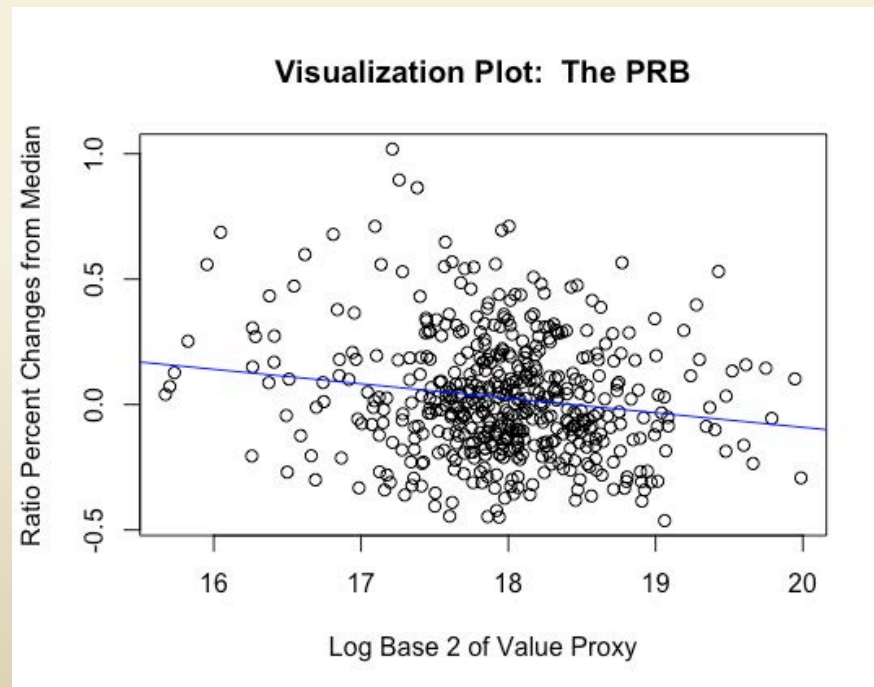
# Interpretation

- Since the PRD is equal to the mean ratio divided by the weighted mean ratio, it should be interpreted as the percentage that the mean ratio is of the weighted mean ratio. It is like saying the central tendency of the ratio distribution, as measured by the mean, is shifted a certain percentage away from the weighted mean. This interpretation, however, is not very practical.
- The PRB has a very practical interpretation: as market value doubles or halves, the ratios change by a certain percentage.



# Visualization

- The PRD does not lend itself to a useful graphical visualization, although one is possible.
- The PRB has a strong graphical visualization because it is based on linear regression with one independent variable.



# Outlier Bias

- The PRB and PRD are compared based on how they handle an incrementally higher number of outliers using a monte-carlo simulation with 10,000 iterations, no vertical inequity, and an average and equal amount of variation in both the Assessed Value and the Sale Price (COD ~ 11).
- Outliers of size 5, 10, and 15 are added to the sample of size 500 in each scenario.



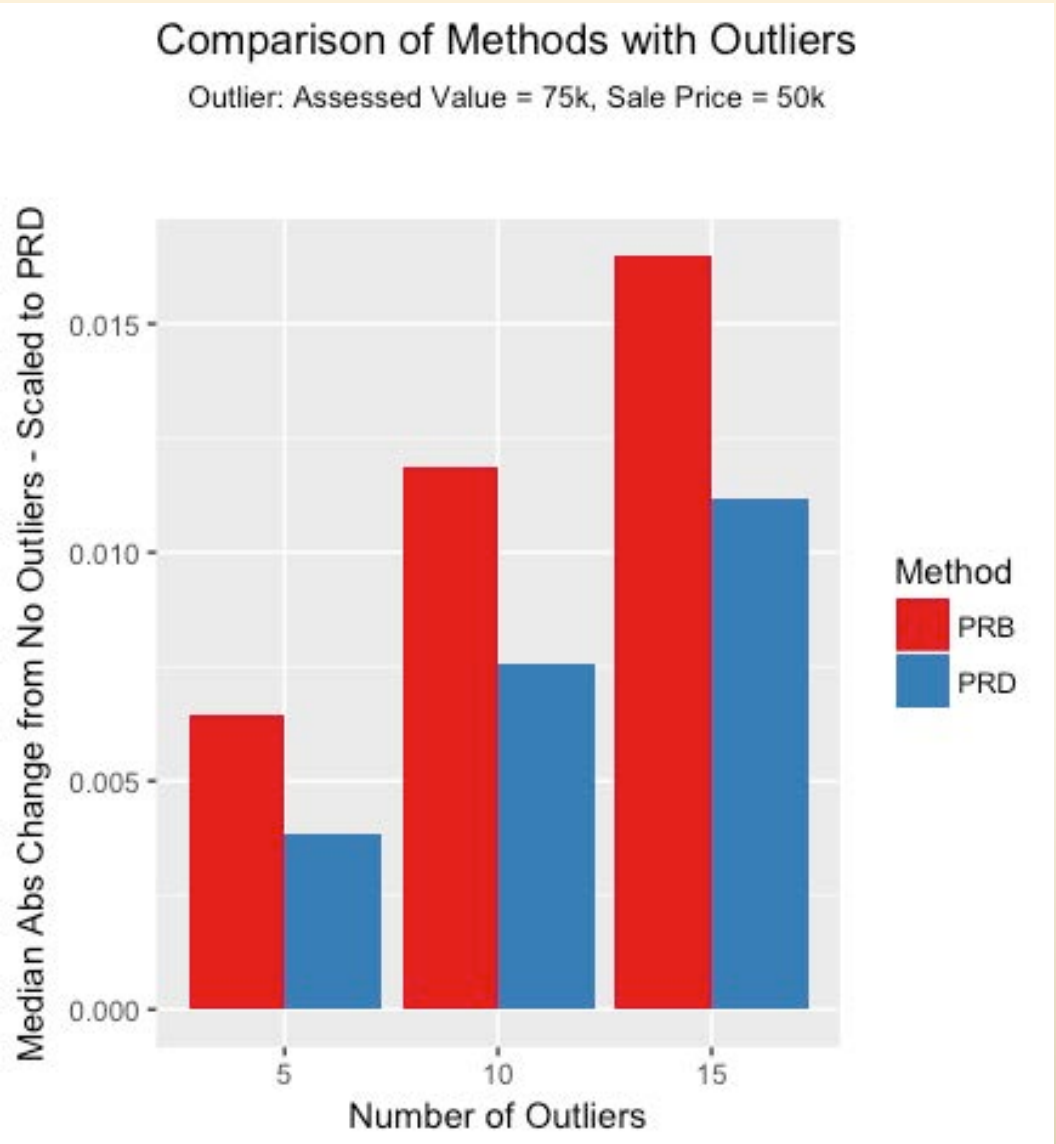
# Outlier Bias

- The median absolute deviation from the method with outliers to the method without outliers is used to show how much affect each number of outliers had.
- The median absolute deviation of the PRB and the PRD are on different scales, so a factor of approximately 0.355 was used on the PRB to equalize the scales.

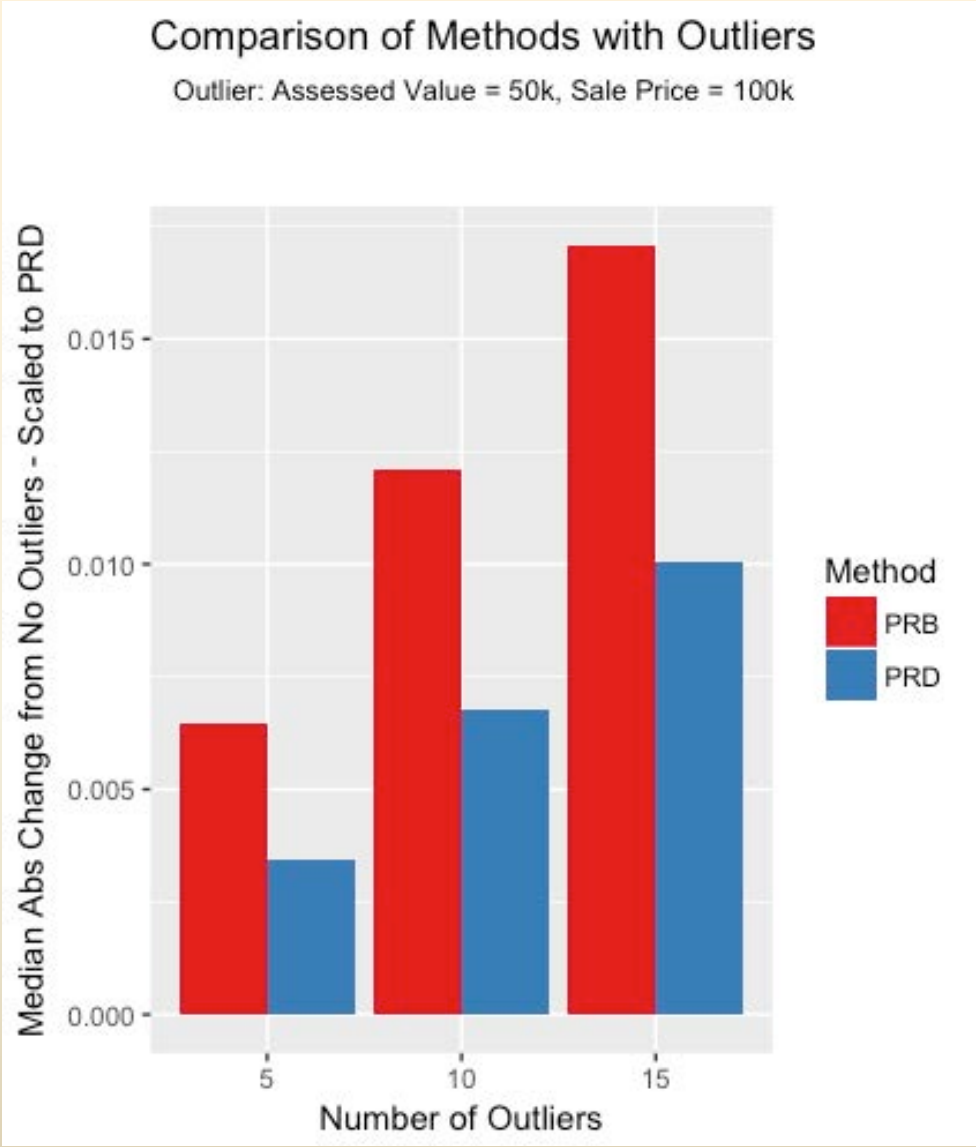




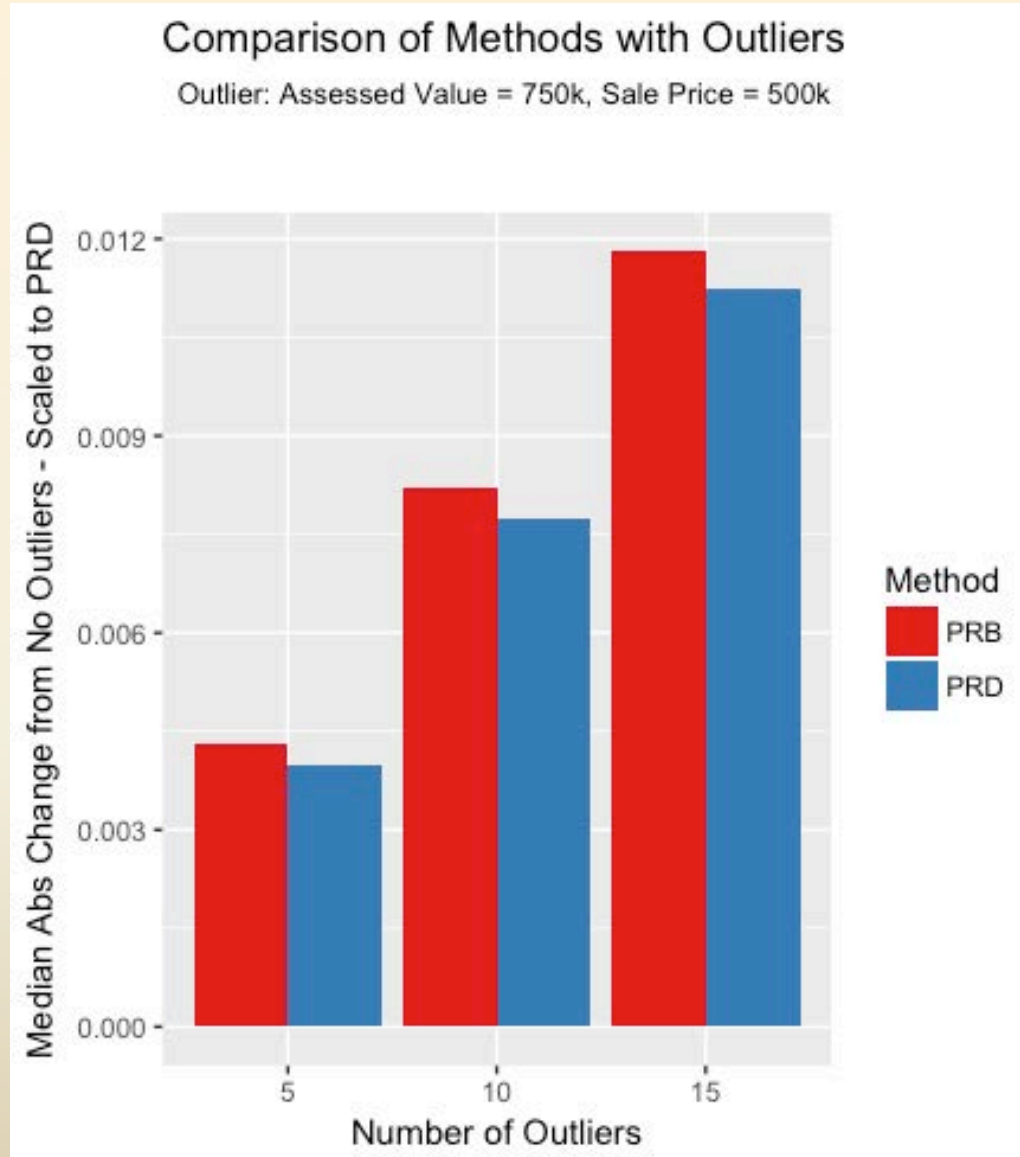
# Outlier Bias – Low Market Values



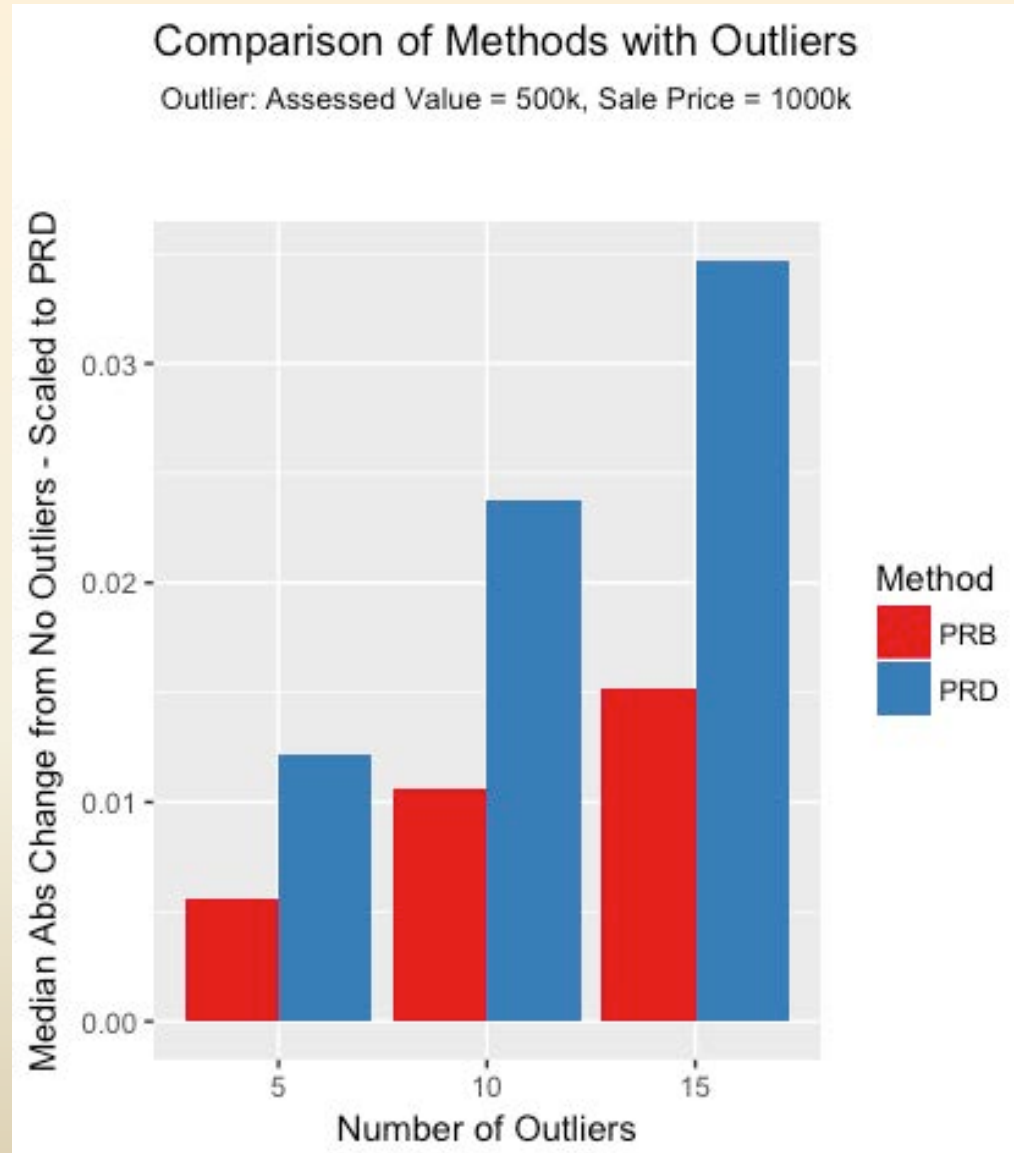
# Outlier Bias – Low Market Values



# Outlier Bias – High Market Values



# Outlier Bias – High Market Values



# Outlier Bias

- Key Conclusions:
  - The PRB is more affected than the PRD when the outlier's market value (assessed values and sale prices) is relatively low.
  - The PRD is more affected than the PRB when an outlier has a sale price that is relatively high.
  - Overall, the PRB is preferable in cases where outlier ratios have relatively high market values, especially in the case of high sale prices, and the PRD is preferable in cases where outlier ratios have relatively low market values.



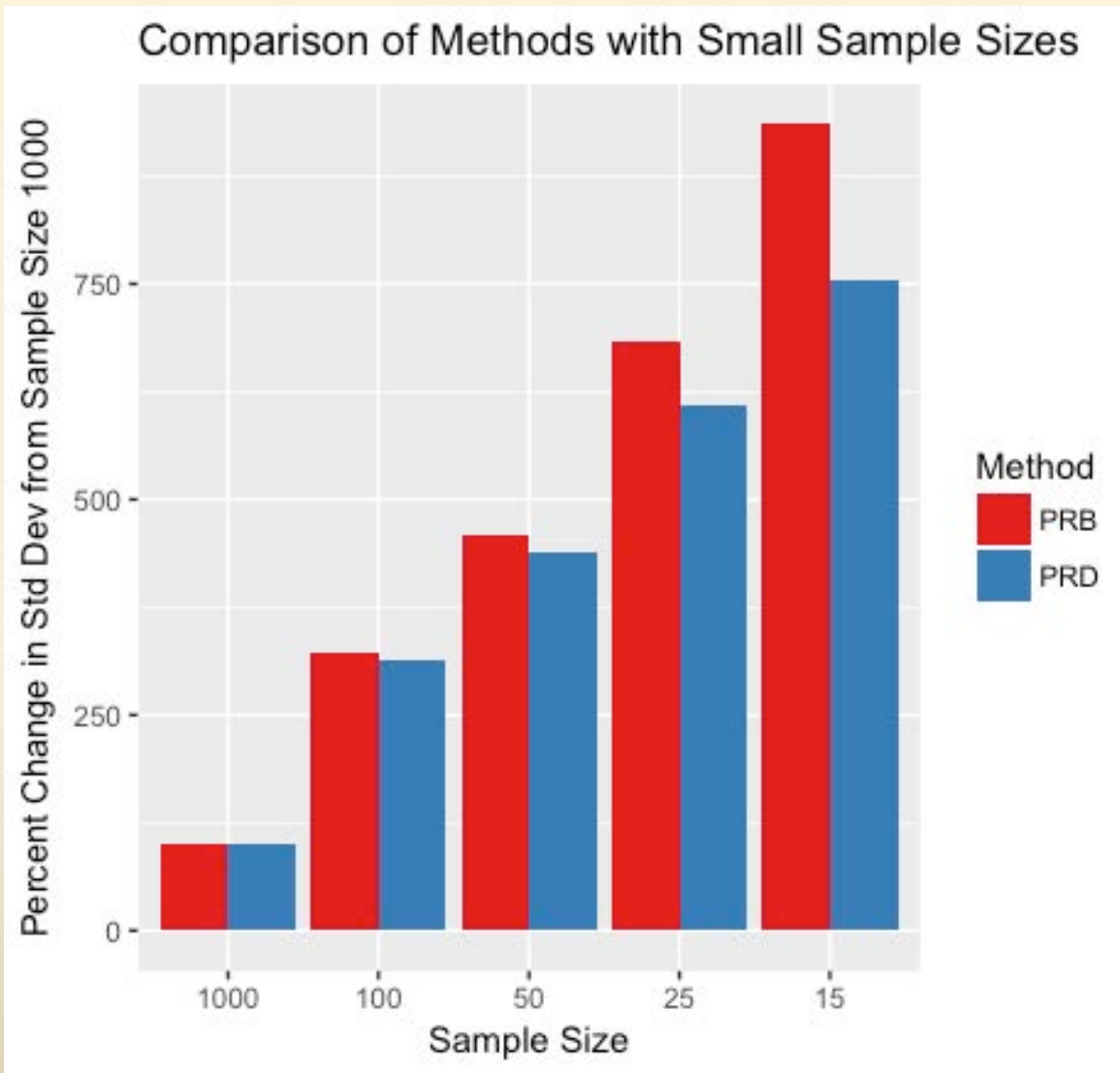


# Small Sample Sizes

- A similar monte-carlo simulation as earlier was performed, with 10,000 iterations, no vertical inequity, and an equal amount of variation in both the Assessed Value and the Sale Price (COD ~ 11). The standard deviation of the PRD and the PRB was calculated for these 10,000 iterations across the range of five sample sizes: 1000, 100, 50, 25, 15.
- The percent change with respect to the standard deviation of the sample size of 1000 was calculated for each sample size. This not only gave a baseline of the standard deviation, but also showed how it relatively changed for each method based on sample size.



# Small Sample Sizes



Conclusion: the PRD shows lower point estimate relative variation increases as sample size decreases.

# Confidence Intervals and Statistical Testing

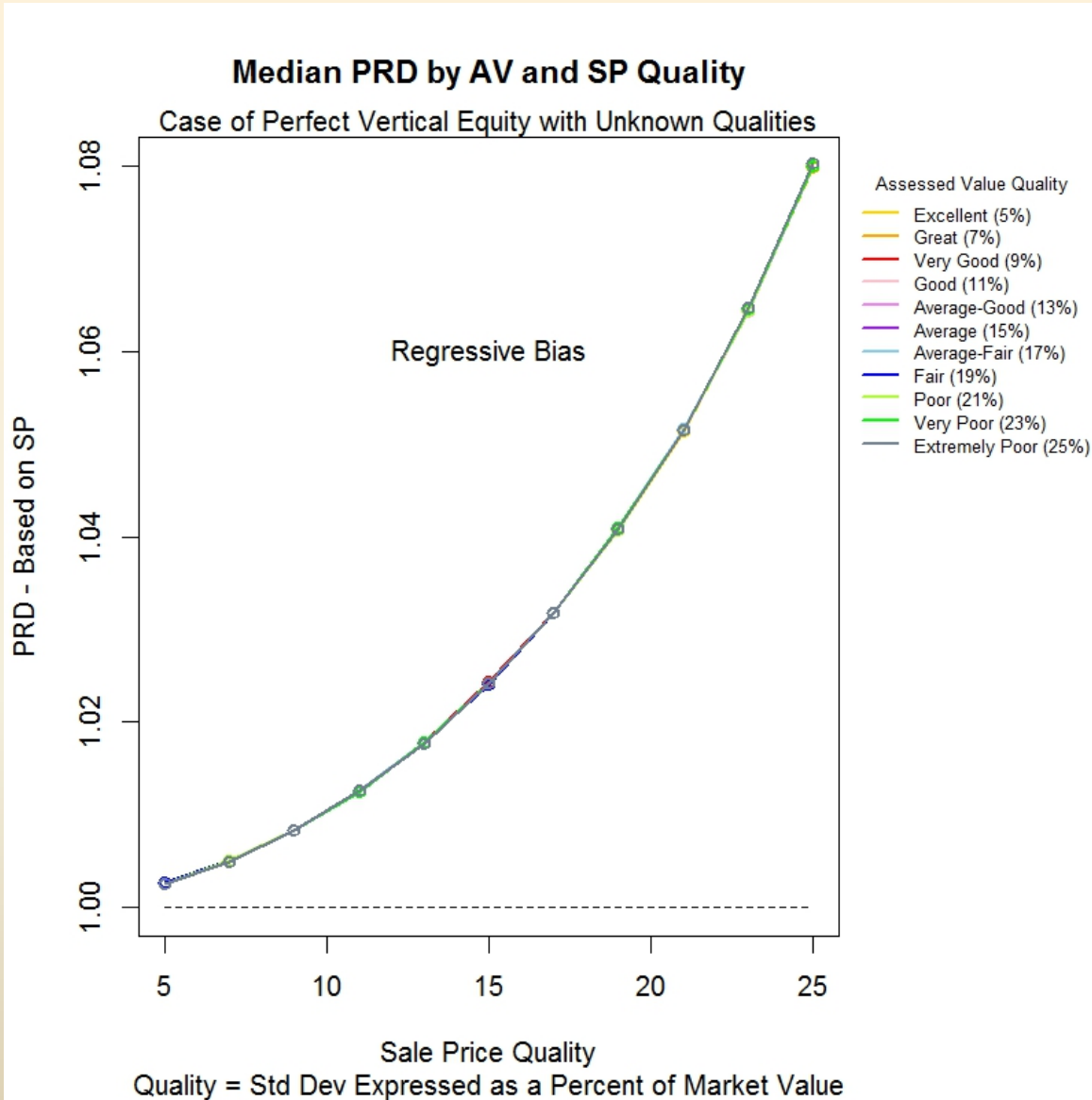
- The PRB naturally has a confidence interval formula because it is based on the regression framework.

$$(B_1 - t_{1-\alpha/2, n-2} * SE_{B_1}, B_1 + t_{1-\alpha/2, n-2} * SE_{B_1})$$

- The PRD does not have a confidence interval formula, but is available by bootstrapping. However, bootstrapping is not easily understood or implemented by most practitioners and the confidence interval bounds could change slightly from run to run.



# Bias Due to False Correlation



## Bias

The PRD is biased toward regressivity in all cases, but the degree to which is determined entirely by the quality of the sale prices.

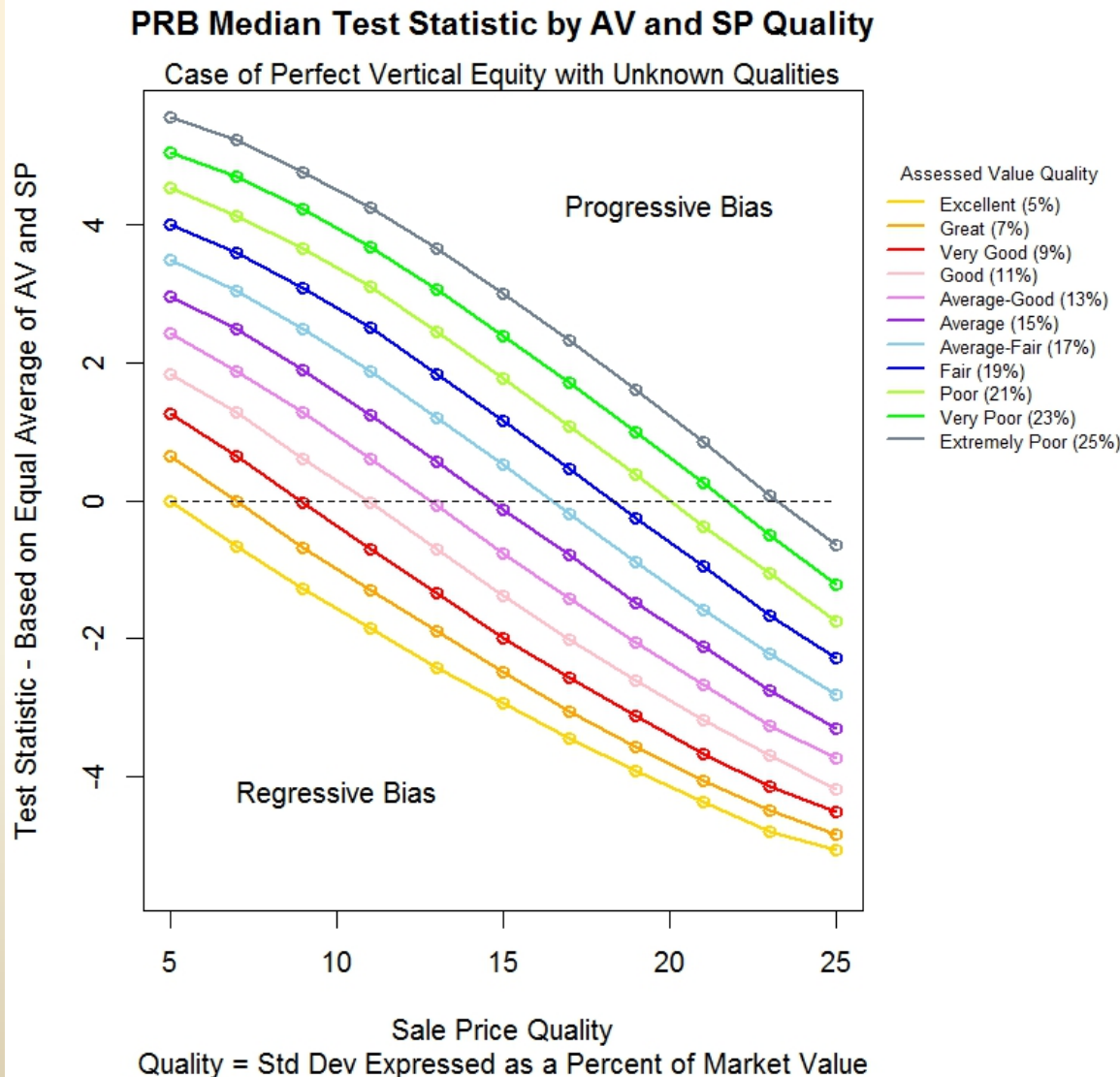
The worse the quality of the sale prices, the more biased the PRD is toward regressivity.

# Bias Due to False Correlation

## Bias

The bias is more toward progressivity the more the assessed value quality is worse than the sale price quality. The bias is more toward regressivity the more the sale price quality is worse than the assessed value quality.

The degree of bias is determined by the difference in quality between the sale price and assessed value.





# The nature of the variation

- The better/worse the quality of the estimate of market value, then the less/more variable the estimate (Assessed Value or Sale Price) will be around the mythical market value.
- So, the amount of variation in the Assessed Value and the Sale Price is dependent on the quality of that particular estimate.



# So, what's the problem?

- As natural variations bring about lower values of the Assessed Value for a given mythical market value, ratios will tend to be lower because the Assessed Value will tend to be lower than the Sale Price. As natural variations bring about higher values of the Assessed Value for a given mythical market value, ratios will tend to be higher because the Assessed Value will tend to be higher than the Sale Price.



# So, what's the problem?

- As natural variations bring about lower values of the Sale Price for a given mythical market value, ratios will tend to be higher because the Sale Price will tend to be lower than the Assessed Value. As natural variations bring about higher values of the Sale Price for a given mythical market value, ratios will tend to be lower because the Sale Price will tend to be higher than the Assessed Value.
- So, aside from any real vertical inequity there is in the ratios, *there is a bias due to the degree of variation in the assessed value and the sale price.*



# So, what's the problem?

- The choice for the market value proxy creates this natural variation with respect to the ratios.
- This bias issue also appears to be similar to the issue of spurious correlation identified by Karl Pearson in his 1897 article “Mathematical Contributions to the Theory of Evolution – On a Form of Spurious Correlation Which May Arise When Indices Are Used in the Measurement of Organs” Proceedings from the Royal Society of London 60: 489 – 498.



# Bias Due to False Correlation

- According to the results of our false correlation bias simulations, recorded earlier:
  - The PRD demonstrates an inherent bias toward regressivity that depends on the quality of the sale prices (their variation around the mythical market value).
  - The PRB demonstrates a bias toward regressivity when the sale prices are of a worse quality than the assessed values and a bias toward progressivity when the assessed values are of a worse quality than the sale prices. However, when the sale price and assessed value are roughly equal in quality, the false correlation bias is minimal.



# Error Rates in Application

- Again, a similar simulation with 1000 iterations from a ratio distribution with COD of 15 was performed and error rates were examined for three cases: no vertical inequity (PRB  $\sim$  0, PRD  $\sim$  1), regressivity (PRB  $\sim$  -0.048, PRD  $\sim$  1.019), and progressivity (PRB  $\sim$  0.043, PRD  $\sim$  0.985).
- Error rates examined were the true positive and false negative error rates for the cases when vertical inequity exists of either type and the false positive and true negative when vertical inequity does not exist.



# Error Rates in Application

- False correlation bias was removed by tweaking the quality of the sale prices and assessed values in the simulation to cancel out that effect because we have already explored that in detail.
- Decisions are made using a 95% one-sided confidence interval.
- Case: No vertical inequity (methods perform similarly)

Measure	True Negative Error Rate	False Positive Error Rate
PRB	90.5%	9.5%
PRD	86.9%	13.1%





# Error Rates in Application

- Case: regressivity (methods perform similarly)

Measure	True Positive Error Rate	False Negative Error Rate
PRB	97.9%	2.1%
PRD	99.3%	0.7%

- Case: progressivity (methods perform similarly)

Measure	True Positive Error Rate	False Negative Error Rate
PRB	97.6%	2.4%
PRD	97.8%	2.2%



# Overall Conclusions

Factor	Edge
Ease of Calculation	PRD
General Assumptions	PRD
Interpretation	PRB
Visualization	PRB
Outlier Bias – Low Market Values	PRD
Outlier Bias – High Market Values	PRB
Small Sample Sizes	PRD
Confidence Intervals and Statistical Testing	PRB
Degree of Bias Due to False Correlation	PRB
Error Rates in Application	Tie



# Can we improve the PRD?

- Let's try changing the market-value proxy used in the traditional PRD to match the market value proxy used in the PRB.
- How would this Alternative PRD do in the Decathlon?
- Price Related Differential (PRD) - Alternate
  - = mean ratio / weighted mean ratio
  - =  $\text{mean}(R) / (\text{mean}(R * \text{Value\_Proxy}) / \text{mean}(\text{Value\_Proxy}))$
  - where  $\text{Value\_Proxy} = 0.5 * (\text{AV} / \text{median}(R)) + 0.5 * \text{SP}$ , as in the case of the PRB, and  $R = \text{AV}/\text{SP}$

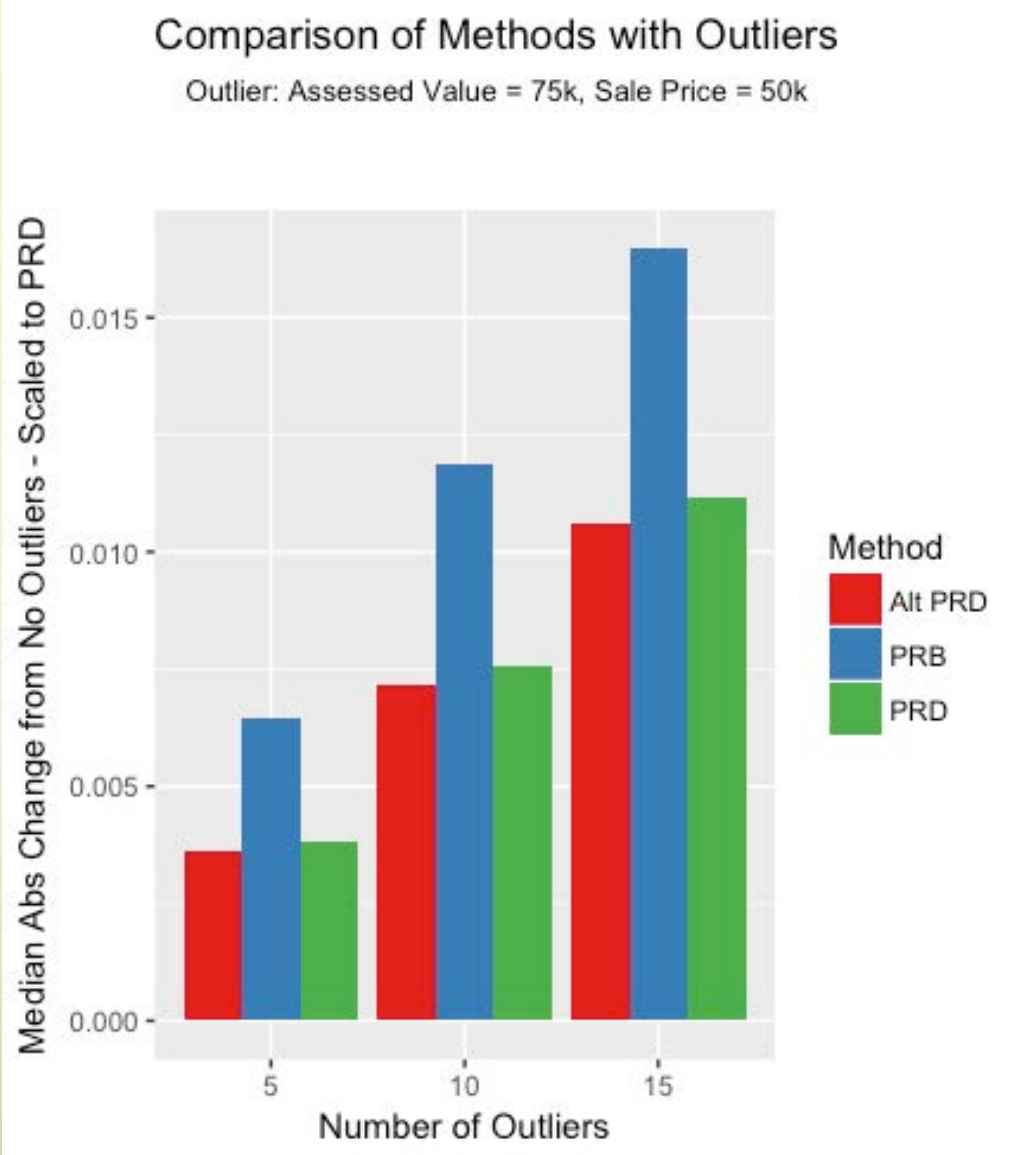


# Vertical Equity Decathlon Round 2

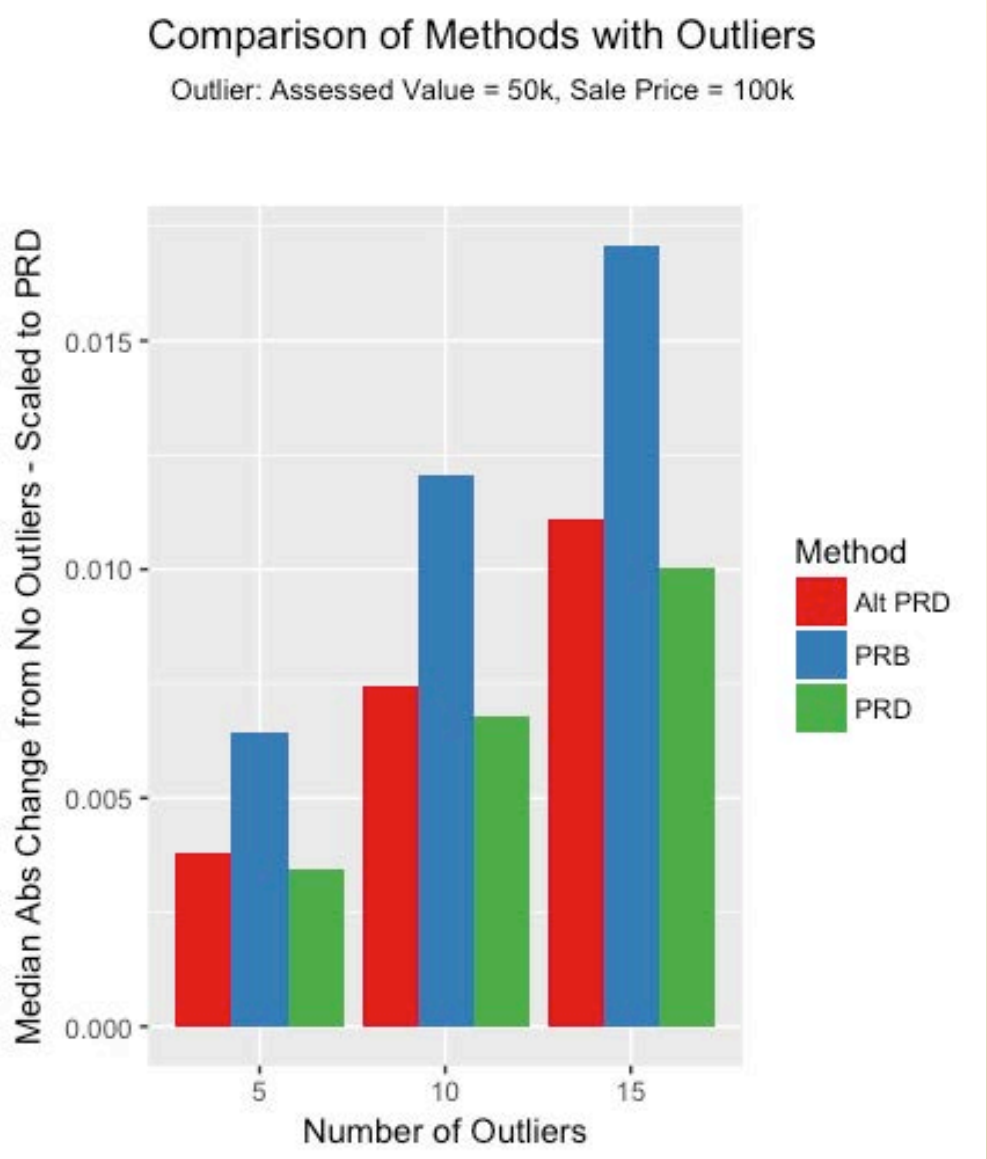
- The following five categories will not change in Round 2 because the basic structure of the methods remain the same:
  - Ease of Calculation
  - General Assumptions
  - Interpretation
  - Visualization
  - Confidence Intervals and Statistical Testing



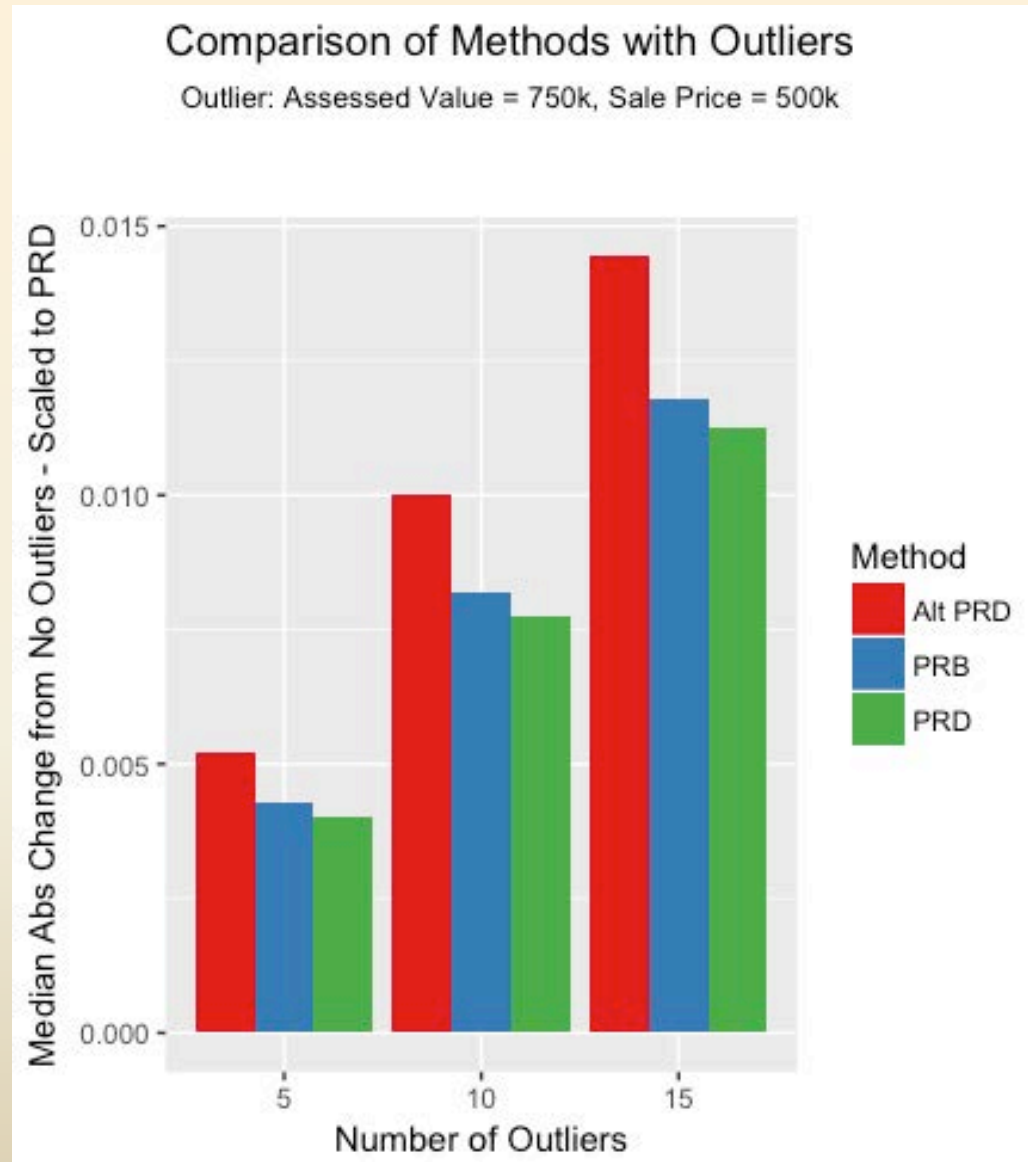
# Outlier Bias – Low Market Values



# Outlier Bias – Low Market Values

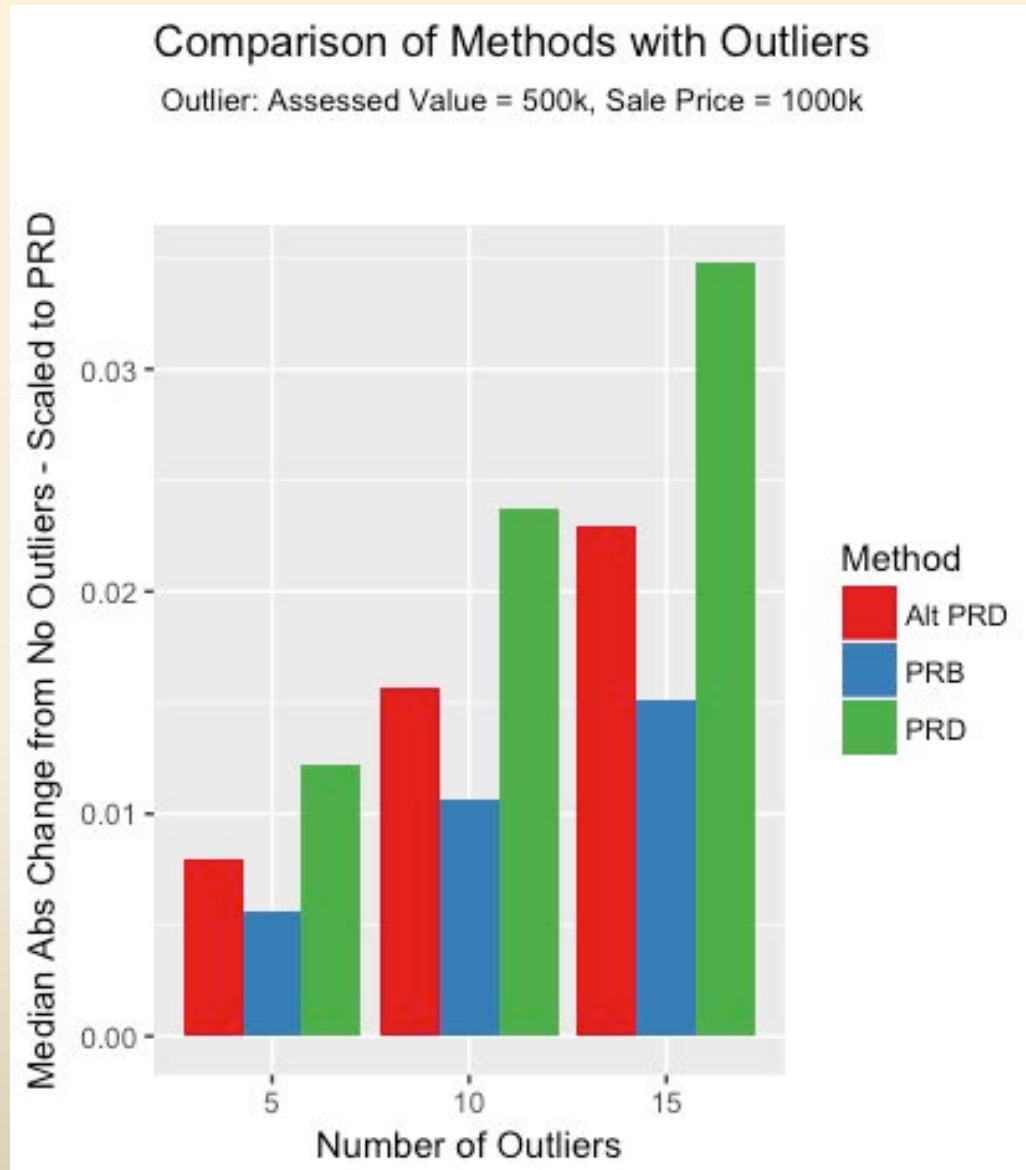


# Outlier Bias – High Market Values





# Outlier Bias – High Market Values



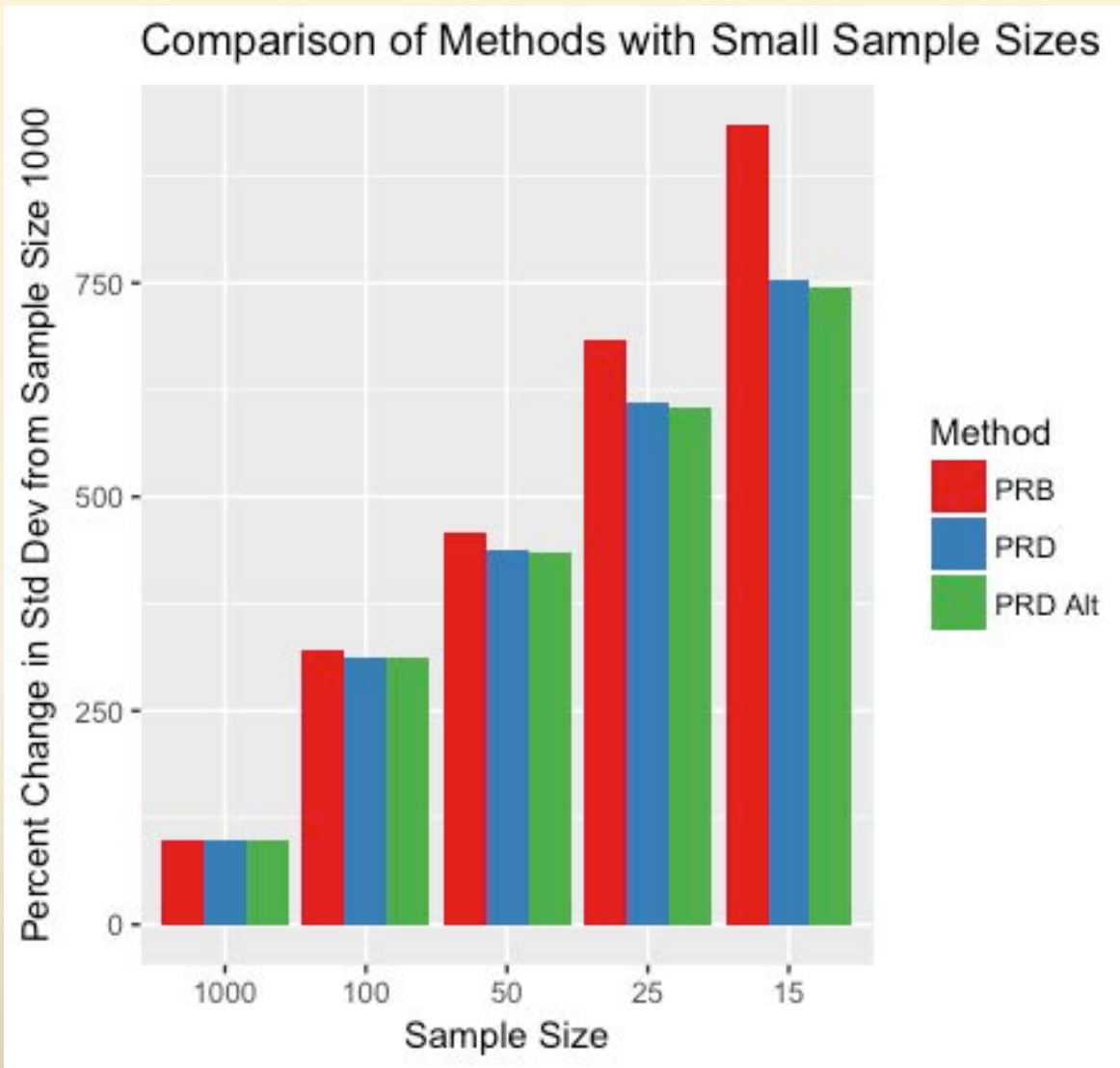
# Outlier Bias

- Key Conclusions:

- The PRB is more affected than both the PRD and the Alternative PRD when the outlier's market value (assessed values and sale prices) is relatively low.
- The Alternative PRD is more affected than the PRB and the PRD when an outlier has a relatively high market value and a larger assessed value than sale price. The PRD is more affected than the PRB and the Alternative PRD when an outlier has a sale price that is relatively high.
- Overall, the PRB is preferable in cases where outlier ratios have relatively high market values and the PRD and Alternative PRD are preferable in cases where outlier ratios have relatively low market values.

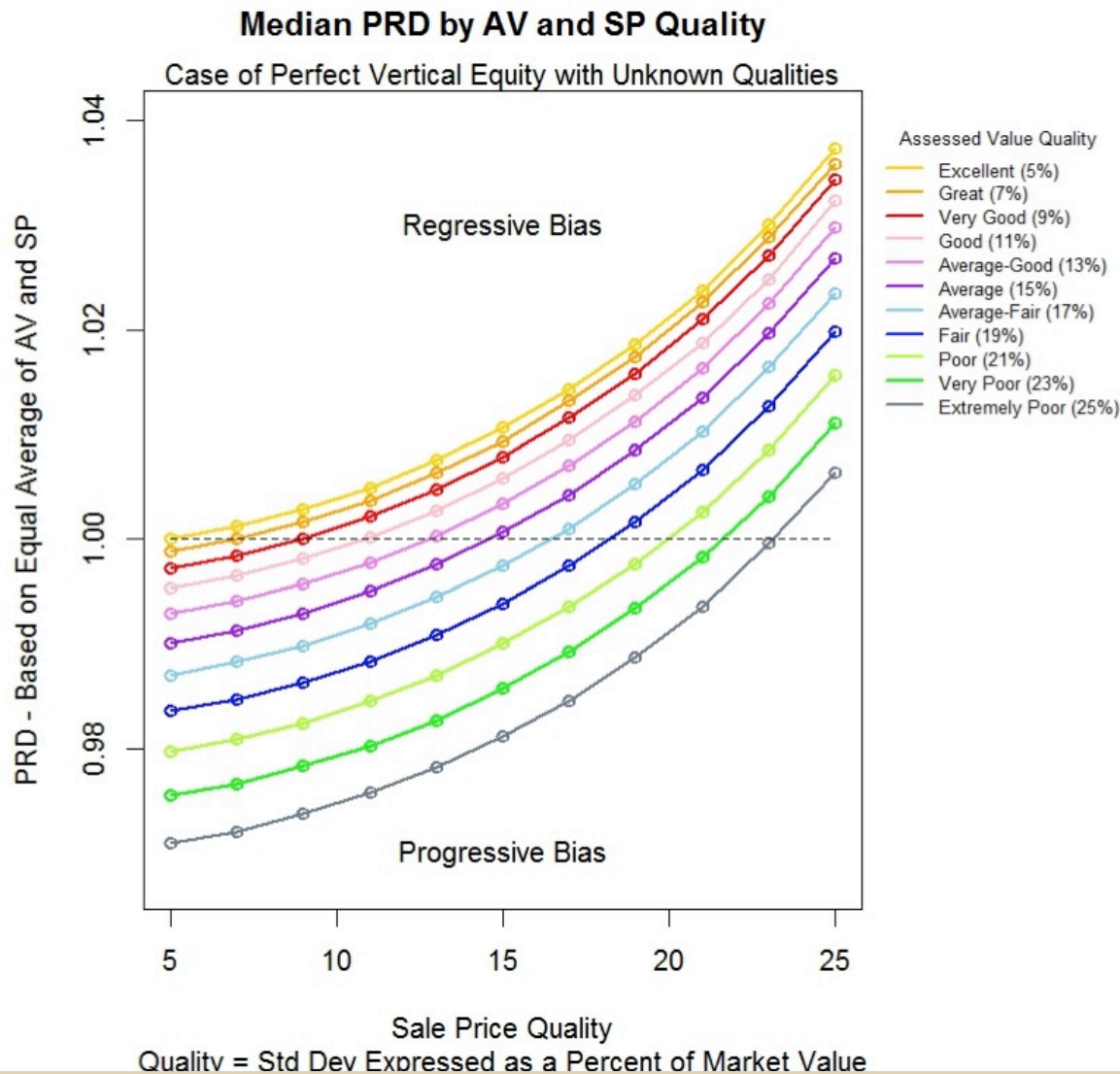


# Small Sample Sizes



Conclusion: the Alternative PRD improved slightly upon the results achieved by the PRD in the first run.

# Bias Due to False Correlation



## Bias

The bias is more toward progressivity the more the assessed value quality is worse than the sale price quality. The bias is more toward regressivity the more the sale price quality is worse than the assessed value quality.

The degree of bias is determined by the difference in quality between the sale price and assessed value.

# Error Rates in Application

- The Alternative PRD is now included and false correlation bias was mitigated in the same way as earlier.
- Case: No vertical inequity (methods perform similarly)

Measure	True Negative Error Rate	False Positive Error Rate
PRB	90.5%	9.5%
PRD	86.9%	13.1%
Alternative PRD	87.2%	12.8%



# Error Rates in Application

- Case: regressivity (methods perform similarly)

Measure	True Positive Error Rate	False Negative Error Rate
PRB	97.9%	2.1%
PRD	99.3%	0.7%
Alternative PRD	98.5%	1.5%

- Case: progressivity (methods perform similarly)

Measure	True Positive Error Rate	False Negative Error Rate
PRB	97.6%	2.4%
PRD	97.8%	2.2%
Alternative PRD	98.4%	1.6%



# Overall Conclusions

Factor	Edge
Ease of Calculation	PRD
General Assumptions	PRD / Alternative PRD
Interpretation	PRB
Visualization	PRB
Outlier Bias – Low Market Values	PRD / Alternative PRD
Outlier Bias – High Market Values	PRB
Small Sample Sizes	Alternative PRD
Confidence Intervals and Statistical Testing	PRB
Degree of Bias due to False Correlation	PRB / Alternative PRD
Error Rates in Application	Tie





# Conclusions

- Statistically, the PRD and the PRB each have their respective strengths and weaknesses. Of the two methods, the PRB has the edge.
- The Alternative PRD is a statistically-improved version of the PRD. The Alternative PRD brings to the PRD the single most important improvement made in developing the PRB. If the PRD is to be kept around as a framework, the Alternative PRD should be used instead.



# Thank you for coming!



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The logo for GIS-Pro 2017 is centered in the upper half of the image. It features a stylized globe with a grid of latitude and longitude lines, rendered in a golden-brown color. The globe is set against a background of a city skyline at dusk or dawn, with buildings illuminated and a soft purple and blue sky. The text "GIS-Pro 2017" is written in a large, bold, teal-colored font across the middle of the logo. Below the main title, the dates "October 23-26, 2017" and the location "Jacksonville, Florida" are written in a smaller, white, sans-serif font.

**GIS-Pro 2017**  
October 23-26, 2017  
Jacksonville, Florida







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